K_5 with center at F. Let G be one of the points of intersection[1] of K_4 and K_5. Draw the line $l_4 = GE$ intersecting[1] the line l_3 at the point H. With H as center and radius HA draw the circle K_6; let J be one of its intersections with the line l_4. With AJ as radius and P as center draw the circle K_7 intersecting[3] the circle K in the points B and B_1. Draw the chords $l_5 = BPB'$ and $l_6 = B_1PB_1'$. These are the required chords. Our construction has required the drawing of six lines l_1, l_2, l_3, l_4, l_5, l_6 and seven circles K_1, K_2, K_3, K_4, K_5, K_6, K_7, the locating of ten necessary points A, B, B_1, D', D, E, F, G, H, J. (If one assumes that the center C is not given, as one might from the statement of the problem, the construction is much more difficult.)

Also solved by George Agins, E. H. Clarke, H. N. Carleton, H. H. Downing, Emanuel Goldfarb, E. D. Grant, Laura Guggenbuhl, R. A. Johnson, Marcia L. Latham, E. W. Martin, F. V. Morley, A. G. Montgomery, H. L. Olson, W. B. Pierce, Arthur Pelletier, Joseph Rosenbaum, Elijah Swift, Charles Schuman, L. G. Weld, C. C. Yen, and the Proposer.

2754 [1919, 73]. Proposed by J. W. LASLEY, JR., University of North Carolina.

Given $\bar{x} = \tan^{-1} \dfrac{z}{\sqrt{x^2 + y^2}}$, $\bar{y} = \tan^{-1} \dfrac{y}{x}$, $\bar{z} = \log \sqrt{x^2 + y^2 + z^2}$, solve for x, y, and z in terms of $\bar{x}$, $\bar{y}$, and $\bar{z}$.

Solution by E. S. Smith, University of Cincinnati

From the given equations, we have $z/(\sqrt{x^2 + y^2}) = \tan \bar{x}$ (1), $y/x = \tan \bar{y}$ (2), and

$$x^2 + y^2 + z^2 = e^{2\bar{z}}. \tag{3}$$

Substituting the value of y from (2) in (1), gives

$$z = x \sec \bar{y} \tan \bar{x}. \tag{4}$$

Substituting the values of y and z from (2) and (4) in (3), we have

$$x = \pm\, e^{\bar{z}} \cos y \cos \bar{x}. \tag{5}$$

Hence, from (2) and (5),

$$y = \pm\, e^{\bar{z}} \sin \bar{y} \cos \bar{x}. \tag{6}$$

From (4) and (6), we have

$$z = \pm\, e^{\bar{z}} \sin \bar{x}. \tag{7}$$

Hence, the result is $x = \pm\, e^{\bar{z}} \cos \bar{x} \cos \bar{y}$, $y = \pm\, e^{\bar{z}} \sin y \cos \bar{x}$, and $z = \pm\, e^{z} \sin \bar{x}$.

Also solved by Marcia L. Latham, E. W. Martin, H. L. Olson, George Paaswell, Arthur Pelletier, C. H. Richardson, and D. L. Stamy.

2755 [1919, 73]. Proposed by J. L. RILEY, Stephenville, Texas.

Every number whose square is the sum of the squares of two consecutive integers is equal to the sum of the squares of three integers of which two, at least, are consecutive.

Solution by Elijah Swift, University of Vermont.

It is well known that the solution of the equation $a^2 + b^2 = c^2$, where a, b, c are relatively prime integers, is given by the formulas, $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$, where m and n

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points corresponding to G exist. The radius most easily specified for which the existence of G can be proved is the radius AF. It would not do, for example, to say "let us take any radius greater than a half of AF" because that does not specify an exact radius and it would need to be proved that any radius which was used was actually "greater than a half of AF". The simplest radius to use for the construction and a logical proof is the radius AF.

[3] Here we have assumed as in previous cases that there are points of intersection, but in this case there are none unless $AP \leqq AJ$. Since AJ is one-third of the required chord BB', when it exists, it follows that $AP \leqq \frac{1}{3}BB'$, which is impossible if AP is greater than one-third of a diameter of K. If $AP \leqq \frac{1}{3}$ of a diameter, there always exists a chord which is trisected at P.